# Mathematics Entry Competencies

Incoming college level students are expected to bring hands-on skills in computation and algebraic manipulation, as well as conceptual knowledge rooted in a deep and profound understanding of numbers and basic geometry. Incoming students are expected to know basic mathematical concepts in computation, algebra and geometry. These are described in some detail in following sections.

In addition, incoming students need to have a comfort level with mathematics so that they approach problems by investigating their nature, asking questions, and revising approaches as they reflect on them. It is as important to understand why a solution works as it is know how one reaches a solution.

Mastery of mathematics at all levels should include the following characteristics:

1. Thinking conceptually and not just procedurally about mathematics. Mathematics is a way of understanding, a thinking process, and not a collection of detached procedures to be learned and applied separately.
2. Using logical reasoning and common sense to work on problems in order to find solutions. Successful students can explain their processes and can check their solutions to see whether their findings make sense.
3. Using experimental thinking and a willingness to investigate the steps used to reach a solution, and recognizing that there are often multiple approaches to solving a problem.
4. Taking risks and accepting that a first or second attempt may result in a wrong answer, but that each attempt is an opportunity to try new approaches toward solving the problem.
5. Understanding that formulas and algorithms in computation, while important and crucial, are only part of the analytical process.

Successful incoming students understand that mathematical problem solving involves logical reasoning. Technology is important and relevant in understanding mathematics, but students should be aware of the limitations of technology and recognize that calculators and computers are tools to assist but not replace the thinking process. Students should understand the basic mathematical terminology and use it appropriately. Students must pay attention to the wording of problems and move with ease between the symbolic representation of a problem and its verbal representation. Students are expected to write with clarity and cohesiveness.

Successful students will also present an orientation toward learning that presents itself as a willingness to work for significant periods of time on a single problem. Persistence is invaluable in the quest for a solution to a problem. Sustained inquiry - engaging in the process for more than a short period of time - is an important part of the process when solving a problem. Oftentimes this process will help foster a deeper understanding, build confidence, and inspire learning.

Successful students demonstrate active participation in the process of learning mathematics by:

1. being willing to experiment with problems that have multiple solution methods;

2. demonstrating an understanding of the mathematical ideas behind the steps of a solution, as well as the solution;
3. showing an understanding of how to modify patterns to obtain different results;
4. showing an understanding of how to modify strategies to obtain different results; and
5. recognizing when a proposed solution does not work, analyzing why, and using the analysis to seek a valid solution.
6. demonstrating an ability to solve multi-step problems with a variety of strategies.

In the sections that follow, entry level competencies have been detailed in areas of numerical computation, algebra and geometry. Examples demonstrating the levels of understanding needed will become a part of this document when they are completed. An asterisk by a number indicates that the competency listed is one that is expected of students who plan to major in METS areas of study.

## I. Numerical Computation

Conceptual understanding of these basic computations, although not explicitly stated (in the examples), is assumed. While technology is useful in helping students explore and enhance their understanding of basic computations, their ability to conceptually understand and perform basic computations without the aid of technology increases the likelihood of success in college level mathematics courses.

Successful students must be able to:

1. Apply mathematical operations to all real numbers in any form (including integers, rational numbers, radicals, and decimals), following the correct order of operations.
2. Calculate the sum, difference, product, and quotient of complex numbers and express the result in standard form.
3. Recognize and generate equivalent forms of fractions, decimals, and percents.
4. Compare and order real numbers, including finding their approximate locations on the number line.
5. Apply laws of rational exponents to real number bases.
6. Recognize and generate equivalent representations (i.e., scientific notation) for very large and very small numbers, and perform mathematical operations on such numerical representations. Move flexibly between scientific notation and expanded form.
7. Compute quantities involving absolute value.
8. Apply the properties of real numbers (including commutative, associative, identity, inverse, and distributive properties).
9. Perform numerical computations involving units of measurement, standard and metric.
10. Communicate accurately using mathematical terminology (e.g., addend, sum, difference, factor, product, divisor, dividend, quotient, remainder, numerator, denominator, exponent, base, radicand, and index).
11. Accurately record symbolic manipulations used in numerical computations, as well as the solutions of numerical computations (e.g., equal signs, inequality symbols, grouping symbols, exponents, subscripts, and solution sets).

12. Communicate accurately using set notation/terminology (e.g., set-builder notation, element of, well-defined, finite/infinite, subset, proper subset, $\varnothing$, cardinal number, equal, equivalent, and interval notation).
13. Estimate numerical computations and judge the reasonableness of the results of these computations.
14. Apply set operations and relations to sets (i.e., union, intersection, complement, and subsets).
15. Represent sets using graphic organizers, including Venn diagrams.

II. Algebra

Successful students are expected to bring a combination of hands-on skill and conceptual understanding of algebra.

1) Successful students know and apply basic algebraic concepts. They:
   a) Add, subtract, multiply, and divide polynomials, rational expressions, and radical expressions.
   b) Divide polynomials.
   c) Apply properties of exponents and radicals.
   d) Factor polynomials (e.g., greatest common factor, grouping, trinomials, difference of squares, sum and difference of cubes).
   e) Simplify polynomials, rational expressions, and radical expressions.

2) Successful students use various appropriate techniques to solve basic equations and inequalities. They:
   a) Solve linear equations and absolute value equations.
   b) Solve linear inequalities and absolute value inequalities.
   c) Solve systems of linear equations and inequalities with two variables, using algebraic or graphical methods.
   d) Solve quadratic equations by factoring, completing the square, and using the quadratic formula.
   e) Solve rational equations.
   f) Solve radical equations.
   g) *Solve nonlinear inequalities. (Recall, an asterisk by a number indicates that the competency listed is one that is expected of students who plan to major in METS areas of study.)
3) Successful students distinguish among expressions, formulas, equations, functions and relations. They know when it is possible to simplify, solve, substitute or evaluate appropriately. In addition, they:
   a) Correctly apply the algebraic language and notation for functions including domain and range.
   b) Compose and decompose functions and find inverses of basic functions.
   c) Identify and compare a variety of functions (e.g., constant, linear, quadratic, cubic, absolute value, exponential and logarithmic functions) and apply the properties of each.
4) Successful students understand the relationship between equations and graphs. They:
   a) Recognize basic forms of the equation of a line and graph the line without technology.

b) Recognize the basic shape of the graph of a quadratic function; find the vertex; calculate and recognize the relationships among the solutions of the related quadratic equation, zeroes of the function and intercepts of the graph.
c) Recognize and sketch the basic shapes of the graphs of the following functions: constant, linear, quadratic, cubic, square root, cube root, absolute value, exponential and logarithmic (without technology).
d) Describe the effects of parameter changes on functions.
e) Describe and sketch the effects of transformations on the graphs of functions.
f) Represent data in a variety of ways (e.g. scatter plot, graph, and table) and select the most appropriate method.

5) Successful students understand algebra well enough to apply it procedurally and conceptually to a range of common problems. Successful students demonstrate the ability to work with formulas and symbols algebraically. They:
   a) Recognize which type of function or expression best fits the context of a basic application.
   b) Use multiple representations to solve problems (e.g. analytic, numerical, and geometric).
   c) Represent algebraically and solve problems that include linear, quadratic, exponential, and logarithmic relationships.
   d) Use mathematics to solve applications from various fields (e.g. rates of change, compound interest, chemical mixture, population growth, and business).
   e) Solve literal equations and formulas for a specified variable.
   f) Communicate accurately using the vocabulary and symbols of algebra.

6) Successful students understand the appropriate use, as well as the limitation, of appropriate technology. They:
   a) Plot relevant graphs.
   b) Use appropriate problem solving methods.
   c) Recognize when the results produced are unreasonable or represent misinformation.

## III. Geometry

Successful students must possess a basic body of knowledge including but not limited to the Pythagorean Theorem, formulas for perimeter, area, volume, and surface area. Successful students demonstrate an understanding of and can explain the mathematical ideas behind the steps of a solution as well as the solution. Successful students recognize when a proposed strategy does not work, analyze why, and use the analysis to seek a valid solution. Successful students understand the appropriate use as well as the limitations of technology.

Successful students must be able to:

1) Apply properties of similarity and congruence.
2) Recognize and apply properties and theorems of parallel lines cut by a transversal.
3) Recognize and apply properties and theorems related to circles.

4) Determine the area and perimeter of plane figures and use the concept of conservation of area.
5) Apply the basic formulas for volume and surface area of solids.
6) Use deductive reasoning to develop and write simple geometric proofs.
7) Use inductive reasoning in problem situations to build a basis for the use of both proof and counter-examples.
8) Apply properties of similarity, particularly related to triangles, to find unknown geometric measurements including angle measurements, lengths of sides, areas, and volumes.
9) Recognize and represent solids and surfaces in three-dimensional space from a two-dimensional representation (e.g. recognize the features of a three-dimensional object: faces, edges, vertices, and shape).
10) Use coordinate geometry to make connections between algebra and geometry.
    a) Describe lines in the coordinate plane using slope-intercept and point-slope form.
    b) Use slopes to describe the steepness and direction of lines in the coordinate plane and to determine if lines are parallel, perpendicular, or neither.
    c) Relate geometric and algebraic representations of lines, segments, simple curves, circles, and *conic sections. (Recall, an asterisk by a number indicates that the competency listed is one that is expected of students who plan to major in METS areas of study.)
    d) Derive and use the formula for distance between two points.
11) Apply the definitions of sine, cosine, and tangent using right triangle trigonometry and *similarity relations. (Recall, an asterisk by a number indicates that the competency listed is one that is expected of students who plan to major in METS areas of study.)
12) *Use trigonometry for examples of algebraic/geometric relationship, including the Law of Sines/Cosines and Trigonometric Identities. (Recall, an asterisk by a number indicates that the competency listed is one that is expected of students who plan to major in METS areas of study.)
13) Describe and represent transformations and symmetries of plane figures.
14) Make connections between analytic, numerical, and geometric methods to solve problems.