

Precalculus

Precalculus is intended to prepare students for fields of study that would require a high level of algebraic and trigonometric reasoning or Calculus. Topics include the foundational principles of functions, the analysis of functions, algebraic reasoning, geometric reasoning, and trigonometry.

I. Foundation of Functions

Students will use multiple representations of different function types to investigate quantities and describe relationships between quantities. Specifically, students will be able to:

- **Use multiple representations of functions to interpret and describe how two quantities change together.**
 - *Identify constraints on quantities and domains*
 - *Distinguish dependent and independent variables*
 - *Identify domains and ranges*
 - *Effectively communicate using function notation.*
- **Measure, compute, describe and interpret rates of change of quantities embedded in multiple representations.**
 - *Identify constant rates of change*
 - *Determine average rates of change*
 - *Be able to estimate instantaneous rates of change*
- **Use appropriate tools and representations to investigate the patterns and relationships present in multiple function types.**
 - *Work effectively with the following functions: linear, quadratic, exponential, logarithmic, rational, piecewise and absolute value*

II. Analysis of Functions

Students will describe characteristics of different function types and convert between different representations and algebraic forms to analyze and solve meaningful problems. Specifically, students will be able to:

- **Create, use and interpret linear equations and convert between forms as appropriate.**
 - *Identify important values (i.e. slope and intercepts) from multiple representations.*
 - *Determine equations of lines given one point and the slope, two points or statements about proportional relationships.*
- **Create, use and interpret exponential and logarithmic equations and convert between forms as appropriate.**
 - *Explain exponential growth as a constant percentage rate of change*
 - *Interpret half-life and doubling time to create decay and growth models*
 - *Recognize similarities and difference between linear and exponential functions*
 - *Recognize the role of “e” as a natural base*
 - *Describe long-term behavior of exponential models*
 - *Apply the inverse relationship between exponential and logarithmic functions*
- **Create, use and interpret polynomial, power and rational functions.**
 - *Recognize how power functions are different from exponential functions*
 - *Determine whether a graph has symmetry and whether a function is even or odd*
 - *Determine end behavior, maximum, minimum and turning points of a graph*

- *Find roots of a function and correctly graph the function*
- *Graph rational functions and find vertical, horizontal and oblique asymptotes*
- **Construct, use and describe transformations, operations, compositions and inverses of functions.**
 - *Describe how the graph of a function can be the result of vertical and horizontal shifts, stretches, compressions, and reflections of the graph of a basic function.*
 - *Perform arithmetic operations with functions and describe the domain*
 - *Create new functions by composing basic functions and describe the domain*
 - *Decompose a composite function into basic functions*
 - *Determine if a function is one-to-one, and if so find the inverse and describe its domain and range*

III. Algebraic Reasoning

Students will identify and apply algebraic reasoning to write equivalent expressions, solve equations and interpret inequalities. Specifically, students will be able to:

- **Use algebraic techniques to simplify expressions and locate roots.**
 - *Solve quadratic equations by factoring, the square root property, completing the square, and the quadratic formula*
 - *Solve quadratic, absolute value, polynomial and rational inequalities*
 - *Perform operations with complex numbers*
 - *Determine complex roots of polynomials*
- **Use algebraic reasoning to simplify a variety of expressions and find roots of equations involving multiple function types.**
 - *Apply properties of exponents and logarithms*
 - *Solve polynomial, radical, rational, exponential, and logarithmic equations*
- **Use rational exponents to express and simplify a variety of expressions and solve equations.**
 - *Factor out common rational powers*
 - *Simplify fractional expressions involving rational exponents*
- **Solve and apply systems of equations and inequalities.**
 - *Set up and solve systems of equations*
 - *Perform matrix operations*
 - *Use matrices to solve systems of linear equations*
 - *Graph systems of inequalities*

IV. Geometric Reasoning

- *Students will use geometric formulas and proportional reasoning to model and solve problems. Specifically students will be able to apply the Pythagorean Theorem*
- *Determine the distance between points in the plane*
- *Find missing lengths or angles in similar triangles*

V. Trigonometry

Students will model and solve meaningful problems using trigonometric functions and their properties. Specifically, students will be able to:

- **Demonstrate an understanding of the properties of angles and of the basic trigonometric functions.**
 - *Understand the definition of radian measure and be able to convert between radians and degrees*
 - *Apply the concepts of radian measure to arc length and area of the sector of a circle*
 - *Apply radian measure to linear and angular velocity*
 - *Interpret sine and cosine as coordinates on a unit circle*
 - *Understand definitions of sine, cosine, tangent, cotangent, secant and cosecant*
 - *Apply right triangle trigonometry in real-world contexts and on the rectangular coordinate system*
 - *Immediately recall the values of $\sin\theta$, $\cos\theta$, $\tan\theta$, $\sec\theta$, $\csc\theta$ and $\cot\theta$ for the special angles*
- **Prove and use trigonometric identities**
 - *Use the Pythagorean identity (and its variations)*
 - *Use double and half-angle identities*
 - *Use angle addition and subtraction formulas to convert and simplify trigonometric expressions*
- **Identify important properties of the graphs of trigonometric functions**
 - *Identify amplitude, period, frequency, phase shift (domain shift) and vertical and horizontal shifts and stretches*
 - *Graph trigonometric functions using the properties of the graph*
- **Solve equations involving trigonometric functions.**
 - *Use identities, properties and factoring to simplify a trigonometric equation*
 - *Find general solutions to a trigonometric equation as well as solutions within a given interval*
- **Solve for missing lengths or angles of oblique triangles.**
 - *Apply the Law of Sines or the Law of Cosines*
- **Use and describe inverse trigonometric functions.**
 - *Use a calculator and reference angle to evaluate inverse trigonometric functions*
 - *Solve equations using properties of inverse trigonometric functions*
 - *Describe domain and range of inverse trigonometric functions*
- **Vectors and Polar Coordinates**
 - *Find the magnitude and direction for the vector, given its initial point and its terminal point*
 - *Find the horizontal and vertical components of a vector, given its magnitude and direction*
 - *Perform vector operations*
 - *Represent vectors in polar form*