

AGENDA ITEM SUMMARY

AGENDA ITEM

Missouri Math Pathways Initiative
Coordinating Board for Higher Education
June 8, 2017

DESCRIPTION

Since October 2014, the Missouri Department of Higher Education has worked closely with the Missouri Mathematics Pathways Taskforce to explore options and make recommendations to increase student success in gateway mathematics courses. As a result, the Taskforce has developed mathematics pathways and co-requisite mathematical models of education for those students who require remedial education or extra assistance.

Background

In September 2014, Missouri was chosen as one of six states to participate in a mathematics pathways initiative, with the intent to increase student success in mathematics gateway courses – which best align with the students’ programs of study – and ultimately increase degree completion. Beginning in October 2014, the Missouri Department of Higher Education, in partnership with Complete College America and the Charles A. Dana Center, has worked closely with the Missouri Mathematics Pathways Taskforce (MMPT) – which is comprised of mathematics faculty representatives from each of the two- year public institutions, the four-year public institutions, and the State Technical College of Missouri – to develop plans and release recommendations to achieve the goals of the Mathematics Pathways Initiative. This includes the Missouri Mathematics Summits, the Missouri Mathematics Pathways Reports, and, most recently, the development of alternative mathematics pathways.

Current Status

After over a year of vigorous and in-depth discussion and debate, and in a show of compromise and cooperation, the MMPT has finalized three alternative pathways and the student learning outcomes (SLOs) for those pathways: a *Statistical Reasoning pathway* (Attachment A), for students whose college and career paths require knowledge of the fundamentals of the collection, analysis, and interpretation of data; a *Pre-Calculus pathway* (Attachment B), for students in fields of study or career trajectories which require a high level of algebraic reasoning or Calculus; and a *Mathematical Reasoning and Modeling pathway* (Attachment C) intended for students whose field of study and future career choices do not require in-depth statistical or algebraic reasoning, or calculus.

These three alternative pathways will be offered at every public two- and four-year institution, with the exception of Truman State University and the State Technical College. In addition, each two- and four-year public institution which offers remedial education will, for those that have not already done so, begin implementing co-requisite mathematics courses which align with the alternative pathways and provide additional support to students.

Next Steps

While three alternative pathways and corresponding student learning outcomes have been finalized, a fourth, the Education pathway for students pursuing a career in education, is still in development. The MMPT decided that more input was needed from school boards, education faculty, and the Missouri Department of Elementary and Secondary Education before finalizing this last pathway.

To facilitate the full implementation of the alternative pathways in fall 2017, MDHE and the MMPT will offer regional meetings around the state of Missouri. The purpose of these meetings is to offer information about and guidance on the implementation of the Missouri Mathematics Pathways Initiative. Individuals who would benefit from these meetings include department chairs, counselors and advisors at the secondary and postsecondary levels, other stake-holders, and the general public. To date, meetings have been tentatively scheduled in September for Springfield and Kansas City. Meetings have been proposed for St. Louis, Cape Girardeau, and Columbia, though those have yet to be finalized.

Finally, in order to measure success and implementation of the initiative, MDHE and the MMPT, in cooperation with the Community College Research Center (CCRC), will identify appropriate data points and metrics. This will allow MDHE to evaluate the efficacy of the Missouri Mathematics Pathways Initiative and the co-requisite model of education. These data will be collected throughout the academic year and be evaluated annually by MDHE.

Conclusion

After many years of planning and discussion, the arduous work of the MMPT is coming to fruition as full implementation of the alternative mathematics pathways begins in fall 2017, along with co-requisite mathematical education offered by participating public institutions. By placing students in mathematics courses that best align with their programs of study, the MDHE hopes to increase rates of student success in gateway mathematics courses and on-time degree completions.

STATUTORY REFERENCES

Section 173.005.2(6), RSMo – Statutory requirements regarding CBHE’s responsibility to identify best practices in remediation and provide oversight of the replication of these best practices by public institutions.

RECOMMENDED ACTION

This is an information item only.

ATTACHEMENT

Attachment A – Student learning outcomes, Statistical Reasoning pathway

Attachment B – Student learning outcomes, Pre-Calculus A+B pathway

Attachment C – Student learning outcomes, Mathematical Reasoning and Modeling

Statistical Reasoning

Statistical Reasoning is a first course in statistics for students whose college and career paths require knowledge of the fundamentals of the collection, analysis and interpretation of data.

Topics include the presentation of interpretation of univariate and bivariate data using graphical and numerical methods, probability, discrete and continuous probability distributions, linear regression, an understanding of good practice in study design, statistical inference, confidence intervals and hypothesis testing. Emphasis is placed on the development of statistical thinking, simulation and the use of technology.

Students should develop an appreciation of the need for data to make good decisions and an understanding of the dangers inherent in basing decisions on anecdotal evidence rather than data. To that end, students will use appropriate data-collection methods and statistical techniques to support reasonable conclusion through the following student learning outcomes.

I. Data Exploration

Students will analyze data using graphical and numerical methods to study patterns and departures from patterns, using appropriate technology as needed. Specifically, students will be able to:

- **Construct and interpret graphical displays of distributions of univariate data.**
 - *Create and interpret dotplots, boxplots, stem and leaf plots and histograms.*
 - *Analyze center, shape and spread, as well as clusters, gaps, outliers and other unusual features.*
- **Summarize distributions of univariate data and compare multiple distributions.**
 - *Compute measures of center (median, mean), measures of spread (range, interquartile range, standard deviation) and measures of position (quartiles, other percentiles and standardized scores).*
 - *Compare groups using back-to-back stem and leaf plots, parallel boxplots and dotplots.*
- **Explore bivariate data.**
 - *Analyze scatterplots for patterns, linearity, and outliers.*
 - *Calculate and interpret the correlation coefficient.*
- **Explore categorical data.**
 - *Create and interpret frequency tables and bar charts.*
 - *Compare distributions of categorical data.*

II. Statistical Design

Students will critically evaluate a data-collection plan to answer a given research question. Specifically, students will be able to:

- **Identify characteristics of good study designs. Understand what conclusions are appropriate for a given design and whether conclusions can be generalized to a larger population.**
 - *Identify the population of interest.*
 - *Determine whether an observational or experimental study is appropriate and feasible.*
 - *Explain the difference between and importance of random selection and random assignment in study design.*
- **Know the elements of planning and conducting an observational study.**
 - *Verify basic elements of statistically valid sample survey.*
 - *Determine when a census or a sample survey is appropriate.*
 - *Identify potential sources of bias in sampling and surveys.*
- **Know the elements of planning and conducting an experimental study.**
 - *Verify basic elements of statistically valid experimental design.*
 - *Explain the purpose of including a control group and blinding in an experiment.*
 - *Identify potential sources of confounding in an experiment.*

III. Probability and Simulation

Students will use probability concepts and simulation. Specifically, students will be able to:

- **Determine and interpret probabilities.**
 - *Interpret a probability as a long-run relative frequency of occurrence.*
 - *Calculate the probability of a specified event in a chance experiment with equally likely outcomes.*
- **Use probability distributions to describe the behavior of discrete and continuous random variables.**
 - *Distinguish between discrete random variables and continuous random variables.*
 - *Compute and interpret the mean and standard deviation of the probability distribution of a discrete random variable.*
 - *Demonstrate an understanding of the mean, standard deviation and shape of continuous probability distributions (uniform, normal and skewed).*
- **Understand distributions.**
 - *Distinguish between the distribution of a sample and a sampling distribution.*

- *Describe the sampling distributions of a sample mean and sample proportion in terms of center, shape and spread.*
- *Explain how these relate to sample size.*
- *Identify when the use of the normal distribution is appropriate.*

IV. Statistical Inference

Students will use statistical models to draw conclusions from data. Specifically, students will be able to:

- **Estimate population parameters including confidence intervals when appropriate.**
 - *Verify that the appropriate conditions have been met.*
 - *Construct one-sample confidence intervals for means and for proportions.*
 - *Construct two-sample confidence intervals for means*
 - *Interpret confidence intervals in context and explain the meaning of the confidence level associated with a confidence interval estimate.*
- **Conduct tests of significance when appropriate.**
 - *Verify that the appropriate conditions have been met.*
 - *Carry out one-sample hypothesis tests for means and proportions.*
 - *Carry out two-sample hypothesis tests for means*
 - *Interpret the meaning of rejection of the null hypothesis and of failure to reject the null hypothesis, in context.*
 - *Demonstrate an understanding of the use of a p-value to reach a conclusion and of the difference between practical significance and statistical significance.*

V. Regression Modeling

- *Determine the equation of the least-squares regression line and interpret its slope and intercept in context.*

Precalculus A and B

Precalculus A is intended to prepare students for fields of study that would require a high level of algebraic reasoning or Calculus. Topics include the foundational principles of functions, the analysis of functions and algebraic reasoning.

I. Foundation of Functions

Students will use multiple representations of different function types to investigate quantities and describe relationships between quantities. Specifically, students will be able to:

- **Use multiple representations of functions to interpret and describe how two quantities change together.**
 - *Identify constraints on quantities and domains*
 - *Distinguish dependent and independent variables*
 - *Identify domains and ranges*
 - *Effectively communicate using function notation.*
- **Measure, compute, describe and interpret rates of change of quantities embedded in multiple representations.**
 - *Identify constant rates of change*
 - *Determine average rates of change*
 - *Be able to estimate instantaneous rates of change*
- **Use appropriate tools and representations to investigate the patterns and relationships present in multiple function types.**
 - *Work effectively with the following functions: linear, quadratic, exponential, logarithmic, rational, piecewise and absolute value*

II. Analysis of Functions

Students will describe characteristics of different function types and convert between different representations and algebraic forms to analyze and solve meaningful problems. Specifically, students will be able to:

- **Create, use and interpret linear equations and convert between forms as appropriate.**
 - *Identify important values (i.e. slope and intercepts) from multiple representations.*
 - *Determine equations of lines given one point and the slope, two points or statements about proportional relationships.*
- **Create, use and interpret exponential and logarithmic equations and convert between forms as appropriate.**
 - *Explain exponential growth as a constant percentage rate of change*

- *Interpret half-life and doubling time to create decay and growth models*
- *Recognize similarities and difference between linear and exponential functions*
- *Recognize the role of “e” as a natural base*
- *Describe long-term behavior of exponential models*
- *Apply the inverse relationship between exponential and logarithmic functions*
- **Create, use and interpret polynomial, power and rational functions.**
 - *Recognize how power functions are different from exponential functions*
 - *Determine whether a graph has symmetry and whether a function is even or odd*
 - *Determine end behavior, maximum, minimum and turning points of a graph*
 - *Find roots of a function and correctly graph the function*
 - *Graph rational functions and find vertical, horizontal and oblique asymptotes*
- **Construct, use and describe transformations, operations, compositions and inverses of functions.**
 - *Describe how the graph of a function can be the result of vertical and horizontal shifts, stretches, compressions, and reflections of the graph of a basic function.*
 - *Perform arithmetic operations with functions and describe the domain*
 - *Create new functions by composing basic functions and describe the domain*
 - *Decompose a composite function into basic functions*
 - *Determine if a function is one-to-one, and if so find the inverse and describe its domain and range*

III. Algebraic Reasoning

Students will identify and apply algebraic reasoning to write equivalent expressions, solve equations and interpret inequalities. Specifically, students will be able to:

- **Use algebraic techniques to simplify expressions and locate roots.**
 - *Solve quadratic equations by factoring, the square root property, completing the square, and the quadratic formula*
 - *Solve quadratic, absolute value, polynomial and rational inequalities*
 - *Perform operations with complex numbers*
 - *Determine complex roots of polynomials*
- **Use algebraic reasoning to simplify a variety of expressions and find roots of equations involving multiple function types.**

- *Apply properties of exponents and logarithms*
- *Solve polynomial, radical, rational, exponential, and logarithmic equations*
- **Use rational exponents to express and simplify a variety of expressions and solve equations.**
 - *Factor out common rational powers*
 - *Simplify fractional expressions involving rational exponents*
- **Solve and apply systems of equations and inequalities.**
 - *Set up and solve systems of equations*
 - *Perform matrix operations*
 - *Use matrices to solve systems of linear equations*
 - *Graph systems of inequalities*

Precalculus B is intended to prepare students planning to pursue a degree in the fields of science, technology, engineering or mathematics as well as other fields that require a high level of algebraic reasoning or would require Calculus. *Precalculus B* includes the *trigonometry* component of a precalculus course. It should be noted that while for some students this might be a terminal course, it is intended to prepare students for higher level mathematics courses. Topics include geometric reasoning and trigonometry.

I. Geometric Reasoning

Students will use geometric formulas and proportional reasoning to model and solve problems. Specifically students will be able to :

- *Apply the Pythagorean Theorem*
- *Determine the distance between points in the plane*
- *Find missing lengths or angles in similar triangles*

II. Trigonometry

Students will model and solve meaningful problems using trigonometric functions and their properties. Specifically, students will be able to:

- **Demonstrate an understanding of the properties of angles and of the basic trigonometric functions.**
 - *Understand the definition of radian measure and be able to convert between radians and degrees*
 - *Apply the concepts of radian measure to arc length and area of the sector of a circle*
 - *Apply radian measure to linear and angular velocity*

- *Interpret sine and cosine as coordinates on a unit circle*
- *Understand definitions of sine, cosine, tangent, cotangent, secant and cosecant*
- *Apply right triangle trigonometry in real-world contexts and on the rectangular coordinate system*
- *Immediately recall the values of $\sin\theta$, $\cos\theta$, $\tan\theta$, $\sec\theta$, $\csc\theta$ and $\cot\theta$ for the special angles*
- **Prove and use trigonometric identities**
 - *Use the Pythagorean identity (and its variations)*
 - *Use double and half-angle identities*
 - *Use angle addition and subtraction formulas to convert and simplify trigonometric expressions*
- **Identify important properties of the graphs of trigonometric functions**
 - *Identify amplitude, period, frequency, phase shift (domain shift) and vertical and horizontal shifts and stretches*
 - *Graph trigonometric functions using the properties of the graph*
- **Solve equations involving trigonometric functions.**
 - *Use identities, properties and factoring to simplify a trigonometric equation*
 - *Find general solutions to a trigonometric equation as well as solutions within a given interval*
- **Solve for missing lengths or angles of oblique triangles.**
 - *Apply the Law of Sines or the Law of Cosines*
- **Use and describe inverse trigonometric functions.**
 - *Use a calculator and reference angle to evaluate inverse trigonometric functions*
 - *Solve equations using properties of inverse trigonometric functions*
 - *Describe domain and range of inverse trigonometric functions*
- **Vectors and Polar Coordinates**
 - *Find the magnitude and direction for the vector, given its initial point and its terminal point*
 - *Find the horizontal and vertical components of a vector, given its magnitude and direction*
 - *Perform vector operations*
 - *Represent vectors in polar form*

Mathematical Reasoning and Modeling

Mathematical Reasoning and Modeling is a terminal course in mathematics for students in the humanities. Given the variety of college and career paths falling within the humanities, this course may be customized to fit the student needs for a particular postsecondary institution. The proposed student learning outcomes/objectives form a basic course framework that will be enhanced by including additional outcomes/objectives, as needed.

The purpose of this course is to provide a comprehensive overview of the skills required to navigate the mathematical demands of modern life and prepare students for a deeper understanding of information presented in mathematical terms. Emphasis is placed on improving students' ability to draw conclusions, make decisions, and communicate effectively in mathematical situations that depend upon multiple factors. To that end, students will develop critical thinking and problem solving skills through the following student learning outcomes.

I. Proportional Reasoning

Students will draw conclusions or make decisions using proportional reasoning. Specifically, students will be able to:

- *Use ratios, proportions, rates, and percentages to explain, draw conclusions, or make decisions.*
- *Use units and unit conversions to explain, draw conclusions, or make decisions.*

Possible content topics: Ratios, proportions, rates, percentages, units, conversions, absolute and relative change, geometric proportions, etc.

II. Statistical Reasoning

Students will read, interpret, analyze, and synthesize quantitative data (e.g., graphs, tables, statistics, survey data, etc.) and make reasoned estimates and inferences. Specifically, students will be able to:

- *Collect and organize data in graphs and tables.*
- *Use descriptive statistics to interpret and analyze quantitative data.*
- *Use probability to interpret and analyze quantitative data.*
- *Communicate statistical findings effectively.*

Possible content topics: Probability, descriptive statistics, visual displays of quantitative information, correlation and causation, etc.

III. Mathematical Modeling

Students will create, apply and use mathematical models to solve problems. Specifically, students will be able to:

- *Describe and contrast linear rate and non-linear rate through verbalization and writing.*
- *Create linear and non-linear functions from quantitative data and explain the results.*

- *Interpret and analyze linear and non-linear functions that model data.*

Possible content topics: Linear functions, exponential functions, scatterplots and best fit lines, financial math, etc.

IV. Additional Topics as Determined by Individual Institutions