# Mathematics Entry Level Competencies for Engineering Program

The need for math skills above the general admission level is required for specific METS fields such as engineering. Students entering engineering programs will encounter courses requiring calculus as prerequisites and Calculus I is viewed as basic preparation for any student pursuing a major in engineering. Students are encouraged to arrive with mathematical skills and concepts to prepare them to successfully enter into a Calculus I course, otherwise the time needed to complete precalculus courses may add an extra semester or more to complete the major. Geometry, College Algebra, and Trigonometry are therefore the core entrance competencies into Calculus I and the engineering programs. Geometry preparation must be at the level of learning associated with a college-track course; basic geometry courses are not sufficient.

The Engineering Workgroup deems the items below to be the fundamental concepts or core entrance competencies that all entering students into engineering should possess in order to successfully matriculate into Calculus I.

**Note: All calculations should be performed without the use of technology (i.e., calculator). Some examples of skills are provided in parentheses.**

1. **Algebra and Real Numbers**
   a. Use symbols and operators to represent ideas and objects and the relationships existing between them.
   b. Understand the relationship between measures of the physical world and be able to convert units in a given problem. (Velocity, distance and time: On a 40-mile car trip to Middletown, NY, you drive the first twenty miles at 40 mph and the last twenty miles at 60 mph. What is your average speed in mph, ft/s, and km/h during the trip?)
   c. Know and apply the following algebraic properties of the real number system: identity, associative, commutative, inverse, and distributive.
   d. Express numbers using scientific notation. (Express 0.004312 in powers of 10). Convert numbers from one base system to another (convert 15 to binary and to hexadecimal notation)
   e. Write a number as the product of factors. (Write 42 as the product of prime factors.)

2. **Radicals and Exponents**
   a. Convert between radical and rational exponent form. (Transform $\frac{1}{\sqrt{x+2}}$ to the rational exponent form $(x+2)^{-\frac{1}{2}}$.)

b. Manipulate algebraic expressions that contain integer and rational exponents. (Simplify $4^{-\frac{3}{2}} \bullet 27^{-\frac{2}{3}}$.)

c. Simplify expressions (Expand and simplify $(a+b\sqrt{c})(d+e\sqrt{c})$)

**3. <u>Algebraic Expressions</u>**

a. Add, subtract, multiply, and divide algebraic expressions. (Using polynomial long division find the remainder when $(x^4 - 5x^3 + 2x^2 - 6x + 1) \div (x^2 + 1)$)

b. Simplify algebraic expressions. (Expand and simplify $(x - 3)(x - 2)(x - 1)$.)

**4. <u>Linear Equations, Inequalities and Absolute Values</u>**

a. Understand the meaning of solutions to linear and rational equations and be able to solve such equations whenever appropriate.

b. Determine the equation of a line. (Find the equation of a straight line passing through the points (2, 1) and (5, 4).)

c. Determine the equation of a line that is parallel or perpendicular to a given line. (Find the equation of a line parallel to the line $2y - 3x = 7$ and passing through the point (1, 2).)

d. Solve a two variable system of linear equations by substitution and elimination. (Use a graph to estimate the point of intersection of the lines $2x + 3y = 7$ and $-x + y = 4$. Verify your result using back substitution.)

e. Solve linear equations and inequalities [graphically and algebraically]. (Solve $5(3 - x) > 2(x - 2x)$ for $x$.)

f. Understand the meaning of solutions to linear and absolute value inequalities. Solve linear equations and inequalities with absolute values. (Solve $|x - 4| \geq 3$ for $x$.)

g. Understand using matrices solutions to linear systems of equations in more 2 than variables and be able to use effective ways to find and express possible solutions.

*h. Understand the concepts of matrices and their inverses (if exist), matrix operations, determinants, and be able to perform required computations. Understand how matrices are used to model and solve system of linear equations and be able to perform required appropriate computations.

** Recommended Topics*

**5. <u>Polynomials, Roots of Polynomials, and Rational Inequalities</u>**

a. Understand the properties and graphs of polynomial functions.

b. Understand the zero factor or zero product property; understand the meaning of zeros of polynomial functions and their connection to the graphs of these functions.

c. Solve for the roots of a polynomial by factoring. (Find the roots of $8x^2 + 31x - 4 = 0$.)

d. Understand the meaning of the Remainder Theorem and its application to evaluating polynomial functions.

e. Understand the meaning of the Factor Theorem and its application to solving polynomial equations.
f. Understand the meaning of solutions to polynomial and rational inequalities and be able to solve such inequalities whenever appropriate.
g. Solve simple polynomial inequalities. (Solve $x^2 + 3x + 6 > x - 4$ for $x$.)
h. Solve simple rational inequalities. (Solve $\frac{x-3}{x+1} < 2$ for $x$.)
i. Understand the importance of the Fundamental Theorem of Algebra, its application to polynomial equations, and its connection to complex numbers.

**6. <u>Functions, Graphs and Graphing</u>**

a. Identify the independent and dependent variables of a function.
b. Evaluate a function at a value. (Given $f(x) = 3x^2 - 2x + 4$, find $f(2a)$.)
c. Determine the domain and range of a real valued function. (Find the domain and range of the real valued function $g(x) = \frac{1}{x^2 - 2}$.)
d. Understand the concept of combining functions arithmetically and by composition and be able to perform these operations and recognize the resulting functions and their properties.
e. Evaluate composite functions. (Given $h(r) = 3r^2$ and $g(s) = 2s$, find $h(a + 2) - g(2a)$.)
f. Understand the concept of piecewise-defined functions and be able to translate this knowledge to their properties and graphs.
g. Graph equations and inequalities. (Sketch a graph of the function $f(x) = 3x^2 - 2x + 7$ for $1<x<5$.)
h. Understand the concept of transformation (e.g., shifting, reflecting, stretching, shrinking) of functions and be able to recognize and apply such knowledge when graphing functions.
i. Transform the graph of a known function. (From the graph of $f(x)$, graph $g(x) = 2f(x) - 3$.)
j. Determine whether a basic algebraic function is invertible and, if so, be able to calculate the function's inverse. Know the relationships between a function and its inverse.
k. Understand the properties and graphs of rational functions and be able to generate appropriate information, including axes, intercepts, intervals of continuity, asymptotes (horizontal, vertical, and oblique), and roots. Be able to graph a rational function showing its salient characteristics without using a calculator using properties of a rational function.
l. Know the general characteristics and shapes of the graphs of polynomial, simple rational (eg. $xy = a$), logarithm, exponential and trigonometric functions.
m. Understand the properties and graphs of parabolas, ellipses, and/or hyperbolas and be able to perform basic related algebraic/graphing operations.

**7. <u>Equations of Quadratic Type and Complex Numbers</u>**

a. Understand the concept of complex numbers and be able to perform operations involving them.
b. Calculate the sum, difference, product, and quotient of complex numbers and express the result in standard form.
c. Understand the process of completing the square of a quadratic expression and its connection to solving quadratic equations and graphing.
d. Solve for real and complex roots using the quadratic formula. (Find the roots of $3x^2 + 2x = -1$.)
e. Solve a system of quadratic equations in two variables by substitution. (Solve the system $y = 3 - x^2$ and $y = 4 + 2x^2 - 2x$.)
f. Understand the relationship between quadratic functions and parabolas, and able to connect such knowledge to quadratic equations.

**8. <u>Logarithmic and Exponential Functions</u>**

a. Understand the meaning of solutions to exponential and logarithmic equations and be able to apply the inverse relationship between exponentials and logarithms to equations involving them whenever appropriate.
b. Apply the properties of logarithms and their relationship to exponentials. Be able to perform operations on logarithms. [ $y = \log_a x, a > 0, a \neq 1$, is the inverse of the function $y = a^x$; $\log_a x \Leftrightarrow a^y = x$ ]. (Evaluate $\log_3 27$.)
c. Know the properties of the logarithmic and exponential functions and use them to simplify logarithmic expressions. (Express as a single logarithm: $0.5\log_{10} x - \log_{10} y$.)
d. Know how to solve simple logarithmic and exponential equations. (Solve the equation $3^{x+4} = 4$ for $x$.)
e. Understand the properties and graphs of logarithmic and exponential functions and be able to evaluate and graph such functions.
f. Understand the meaning of exponential growth and decay and apply the knowledge of exponential and logarithmic functions model to applications.

**9. <u>Analytic Geometry</u>**

a. Know and apply the distance formula between two points. (Find the distance between the two points A(1, 2) and B(–5, –3).)
b. Understand the geometric concepts of angle (e.g. initial side, terminal side, coterminal angles, degree, radian, central angle, circular arc length, circular sector area, and reference angle) and be able to apply appropriate properties.
c. Know and apply the circumference and area formulas for circles, triangles, and rectangles. (If you double the radius of a circle, what happens to its circumference?)
d. Know and apply the surface and volume formulas for cylinders, spheres and rectangular solids.
e. Know the relationship between similar triangles. (A rectangle with base $x$ and height 5 is inscribed in an isosceles triangle with base 10 and height 20. Determine $x$.)

f. Know and apply the Pythagorean Theorem to simple geometric problems. (Given a rectangle that is 4 ft by 7 ft determine the length of the diagonal.)

**10. Use of Mathematics to Solve Application from Various Fields**

a. Apply the acquired understanding and knowledge of functions to model appropriate real-world situations and draw mathematical conclusions.
b. Understand the underlining principle of variation and how it is used to model many applications.
*c. Understand the meaning of solutions to systems of nonlinear equations and be able to use effective ways to find and express possible solutions.
d. Understand the meaning of compound interest and apply the knowledge of exponential functions to model this application.
*e. Be able to use trigonometry to model and solve basic applied problems.

** Recommended Topics*

**11. Trigonometric Functions & Their Inverses**

a. Define each of the 6 trigonometric functions ($\sin\theta$, $\cos\theta$, $\tan\theta$, $\cot\theta$, $\sec\theta$, and $\csc\theta$) in terms of the sides of a right triangle. ($\cos\theta = \frac{x}{r}$ where *x* is the adjacent side and *r* is the hypotenuse.)
b. Define each of the 6 trigonometric functions in terms of $\sin\theta$ and $\cos\theta$. ($\tan\theta = \frac{\sin\theta}{\cos\theta}$.)
c. Understand the concepts of the six trigonometric functions, both in terms of a unit circle and a right triangle, and be able to apply such knowledge.
d. Know the domains and ranges for the sine, cosine, and tangent functions; know why domains of inverse trigonometric functions have the usual restrictions (example: solve for x $\sin x = -\frac{\sqrt{3}}{2}$).
e. Convert angle measures between degrees and radians. (Write 120 degrees as a radian measure.)
f. Memorize and use the 30/60/90 and 45/45/90 degree reference triangles.
g. Understand the graphs of the six trigonometric functions and be able to recognize and apply such knowledge (including incorporation of appropriate transformations: shifting, reflecting, stretching, and shrinking).

**12. Trigonometric Identities and Equations**

a. Understand the general nature of proving trigonometric identities and be able to perform such task appropriately.
b. Know and apply the trigonometric identity $\sin^2\theta + \cos^2\theta = 1$. (Simplify the expression $2\cos^2\theta + \sin^2\theta - 1$.)
c. Understand the general nature of trigonometric equations and be able to solve such equations whenever appropriate ($2\sin^2\theta - \sin\theta - 1 = 0$ over $0 \le \theta < 2\pi$).

d. Be familiar with useful formulas (e.g. addition and subtraction, double-angle, half-angle, product-to sum, sum-to-product, law of sines, law of cosines, and Heron's) and able to use them appropriately.
e. Understand the concepts and graphs of inverse trigonometric functions and their related properties, and be able to perform appropriate operations.
f. Understand the trigonometric form and its geometric interpretation for complex numbers, and be able to recognize and perform basic conversions.
g. Understand the multiplication and division of complex numbers in trigonometric form and their respective geometric interpretation.
h. Understand De Moivre's Theorem and its geometric interpretation, and be able to apply the concept to find roots of complex numbers.
i. Understand the basic concepts and operations of two-dimensional vectors, their respective geometric interpretation, and the trigonometric aspect of the inner (dot) product.
j. Understand the geometry of complex numbers.